COMMUNICATIONS TO THE EDITOR

Drainage and Withdrawal of Liquid Films

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When a vessel filled with liquid is emptied, a thin film of liquid clings to the inside of the vessel. This film gradually drains down under the influence of gravity. When we neglect inertial forces, the rate of flow of liquid across a unit breadth is

$$q = \frac{1}{3} \frac{\rho g}{\mu} h^3 \tag{1}$$

and therefore the film thickness h varies at a rate

$$\frac{\partial h}{\partial t} = - \frac{\partial q}{\partial x} = - \frac{\rho g}{\mu} h^2 \frac{\partial h}{\partial x} \tag{2}$$

A film with constant thickness satisfies this relation

$$\frac{\partial h}{\partial t} = \frac{\partial h}{\partial x} = 0 \tag{3}$$

Another solution is the parabolic profile

$$h = \sqrt{\frac{\mu}{\rho g} \frac{x}{t}} \tag{4}$$

This last equation is the generally accepted solution for the profile of the film in a draining vessel when inertial and capillary effects can be neglected. See, for example, Jeffreys (*5*), van Rossum (*7*), Bird, Stewart, and Lightfoot (*1*, problem 2 R), Wallis (*8*, example 6.1), and Chase and Gutfinger (*2*). This solution satisfies the boundary condition at the top of the film

$$h = 0, \quad x = 0 \tag{5}$$

The thickness of this film at the junction with the level of the bath is

$$h = \sqrt{\frac{\mu}{\rho g} \frac{l}{t}} = \sqrt{\frac{\mu v}{\rho g}} \tag{6}$$

when v is the lowering speed of the bath. In reality, this thickness cannot be realized at the level of the bath because of secondary flows caused by the falling level. The liquid in the bath is split into one fraction which stays on the wall and another fraction which flows away from the wall under the advancing level of the bath.

When only gravitational and viscous forces contribute and when inertial and capillary forces can be neglected, Groenveld (*3*) found for the film thickness during withdrawal of a plate from a bath

$$h = 0.66 \sqrt{\frac{\mu v}{\rho g}} \quad Ca = \frac{\mu v}{\sigma} > 0.5, \quad Re = \frac{\rho v h}{\mu} < 1 \tag{7}$$

This thickness is also realized at the liquid level during drainage of a vessel. Only the relative velocity of the plate (wall) and of the bath level is relevant, not their absolute velocity. The profile of the film satisfying the boundary conditions at the top [Equation (5)] and bottom [Equation (7)] of the film is shown in Figure 1. Jeffreys' parabolic solution for the profile holds as long as

$$h < 0.66 \sqrt{\frac{\mu v}{\rho g}} \text{ or when } \frac{x}{l} < (0.66)^2 = 0.44 \tag{8}$$

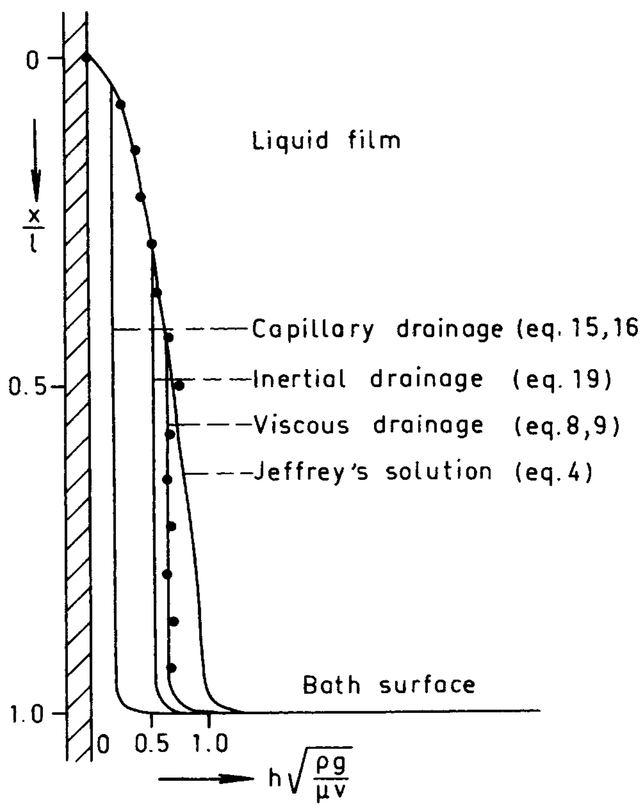

Fig. 1. Draining films for the three theories, with measurements for viscous drainage.

The constant thickness solution holds for the lower part of the film

$$0.44 < \frac{x}{l} < l \tag{9}$$

A certain film thickness propagates downward with the following velocity [Equation (2)]:

$$\left(\frac{\partial x}{\partial t}\right)_h = -\frac{\left(\frac{\partial h}{\partial t}\right)_x}{\left(\frac{\partial h}{\partial x}\right)_t} = \frac{\rho g h^2}{\mu} \tag{10}$$

The junction between the parabolic profile and the constant thickness film therefore propagates downward with the following velocity

$$v_{\text{junction}} = \frac{\rho g}{\mu}(0.66)^2 \frac{\eta v}{\rho g} = 0.44\ v \tag{11}$$

When the liquid level reaches the bottom of the tank, the part of the film with constant thickness will drain down. At a time

$$t = (1 - 0.44)\frac{l}{v} = 0.56\frac{l}{v} \tag{12}$$

and after stagnation of the level, the parabolic solution of Jeffreys will hold for the whole film.

The same holds of course for a plate with length l being withdrawn and removed from a bath. As long as the plate is in contact with the bath, the top half of the film will have a parabolic profile and the lower half a constant thickness. After removal, however, the parabolic part expands at the cost of the constant thickness part. A film profile photographed on a plate being withdrawn from a bath filled with sugar syrup is shown in Figure 1 (v = 11.4 cm./sec., l = 39 cm., μ = 25 poise, ρ = 1.38 g./cc., σ = 70 dyne/cm.). It corresponds well with the theory.

The foregoing holds only when the capillary and inertial forces can be neglected

$$\frac{\mu v}{\sigma} > \frac{1}{2}, \quad \frac{\rho v h}{\mu} < 1 \tag{13}$$

The film thickness at the transition to the bath level is different when inertial or capillary forces contribute. The distribution of the liquid under those conditions will be calculated in the next section. No experimental results are available with which to compare the theories.

For low speed withdrawal of thin liquids, the surface tension has to be taken into account. The curvature of the interface is highest at the transition to the bath. Therefore capillary forces will have to be accounted for there and not along the film where the interface curvatures are small. The surface tension forces lower the thickness of the film remaining on the wall when the liquid level passes (*6*)

$$h = 0.94\left(\frac{\mu v}{\sigma}\right)^{1/6}\sqrt{\frac{\mu v}{\rho g}} \quad \text{for} \quad \frac{\mu v}{\sigma} < 10^{-2} \tag{14}$$

In this case a smaller part of the film is parabolic

$$\frac{x}{l} < 0.9\left(\frac{\mu v}{\sigma}\right)^{1/3} \tag{15}$$

and a larger part of the film has a constant thickness

$$0.9\left(\frac{\mu v}{\sigma}\right)^{1/3} < \frac{x}{l} < 1 \tag{16}$$

The solution of Jeffreys will hold for the whole film at times after removal longer than

$$t = \left(1,11\left(\frac{\sigma}{\mu v}\right)^{1/3} - 1\right)\frac{l}{v} \tag{17}$$

The thickness in high speed withdrawal was calculated by Groenveld (*4*)

$$h = 0.52\sqrt{\frac{\mu v}{\rho g}} \quad \text{for} \quad \frac{\rho v h}{\mu} > 50 \tag{18}$$

As long as the film is in contact with the bath, the profile of the film is now

$$\begin{aligned} &\frac{x}{l} < 0.27 \qquad h = \sqrt{\frac{\mu v}{\rho g}\frac{x}{l}} \\ &0.27 < \frac{x}{l} < 1 \quad h = 0.52\sqrt{\frac{\mu v}{\rho g}} \end{aligned} \tag{19}$$

From the foregoing we may conclude that the film thickness just above the level of the bath during viscous draining of a vessel as predicted by Jeffreys is 52% too high. The amount of fluid clinging to the solid is always minimized when the time available for withdrawal and post withdrawal drainage is used exclusively for withdrawal.

This is relevant for minimizing the water pollution caused by rinsing. A conclusion pertaining to dip-coating may also be drawn from the above results. The fluid film has a more even thickness upon removal from the bath when the capillary forces are important. Low speed withdrawal of thin liquids therefore gives a more even film than high speed viscous withdrawal. But, after waiting long enough after removal, all profiles become parabolic, thus satisfying the theory of Jeffreys.

NOTATION

- g = gravitational acceleration, sq.cm./sec.
- h = film thickness, cm.
- l = distance from film top to bath surface, cm.
- q = liquid flux per unit time and width, sq.cm./sec.
- t = time, sec.
- v = velocity of solid support relative to the bath level, cm./sec.
- x = coordinate, distance from the top of the film, cm.

Greek Letters

- μ = viscosity, poise
- ρ = density, g./cc.
- σ = surface tension, dyne/cm.

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